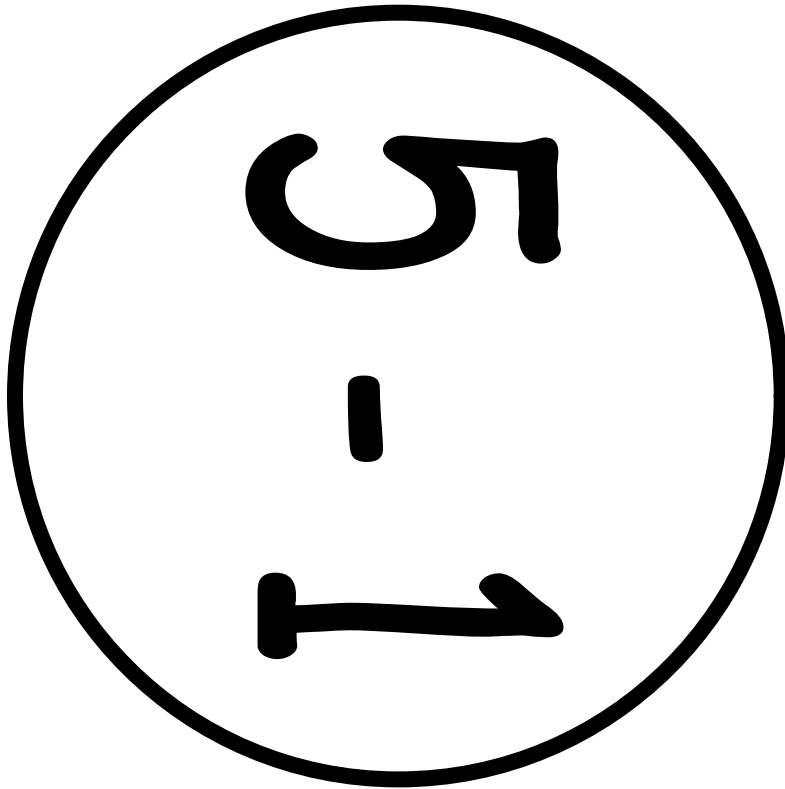


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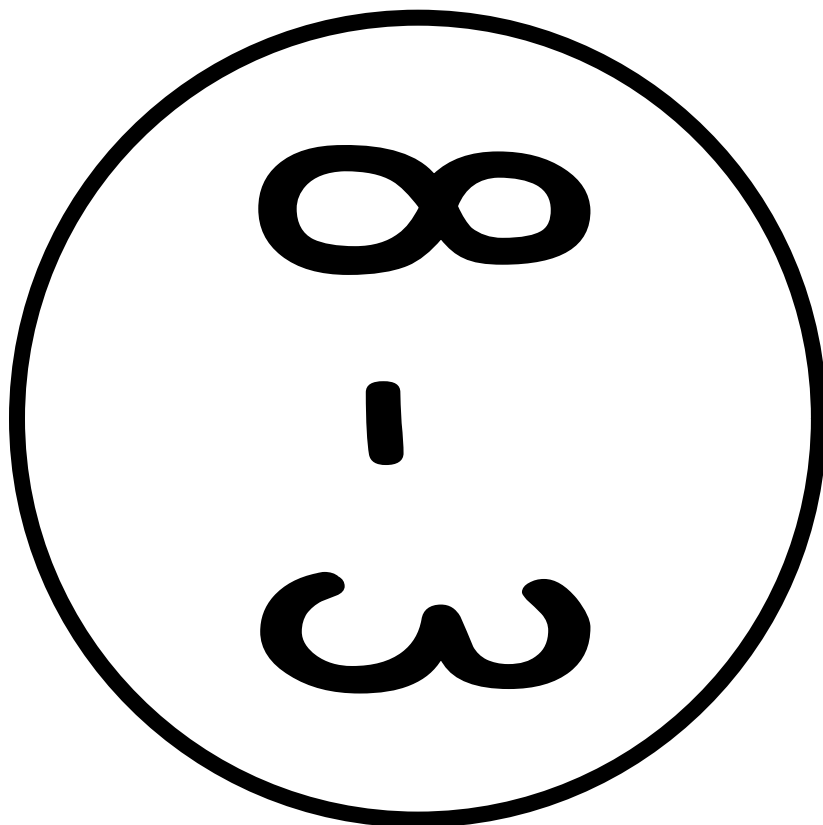
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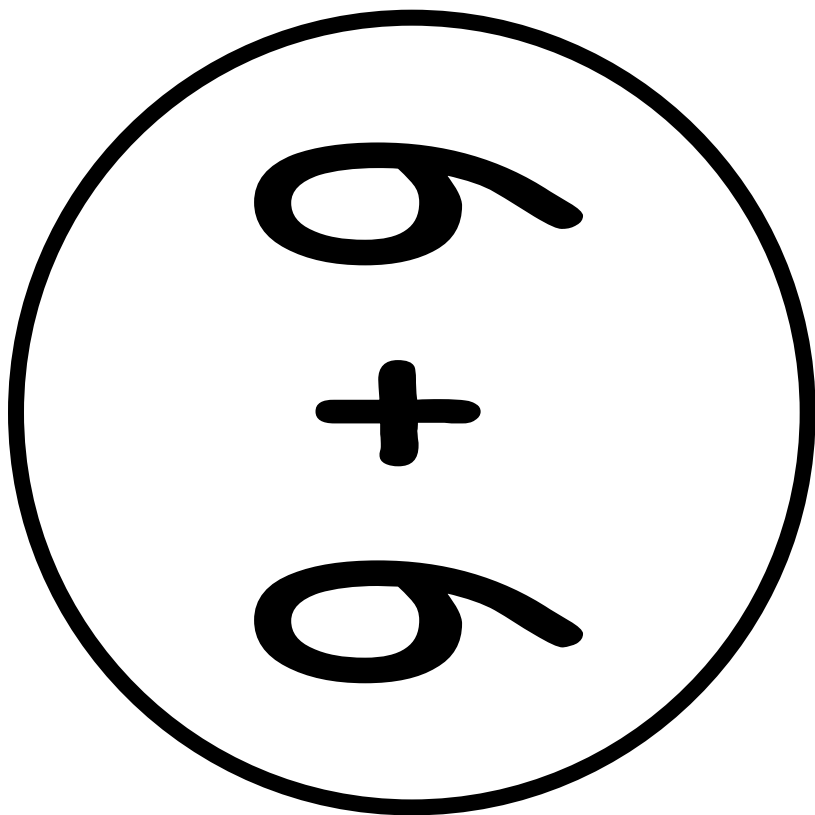


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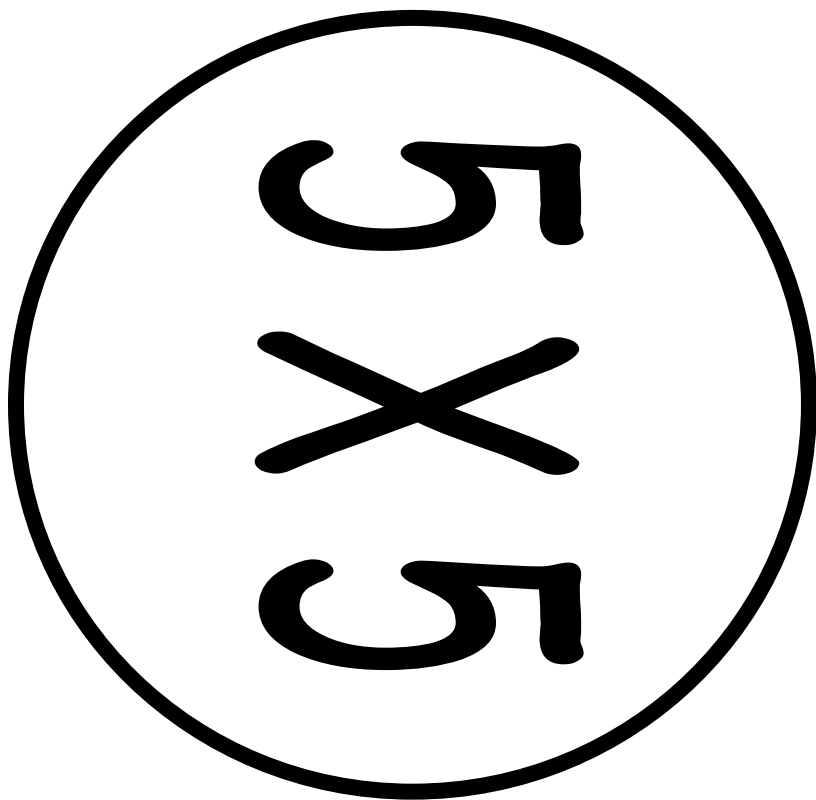


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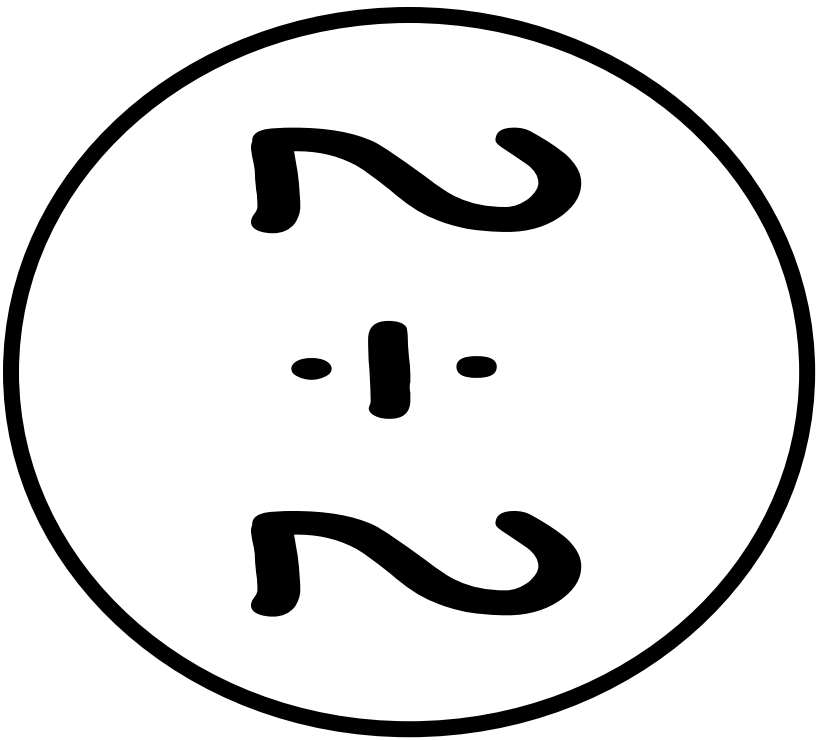
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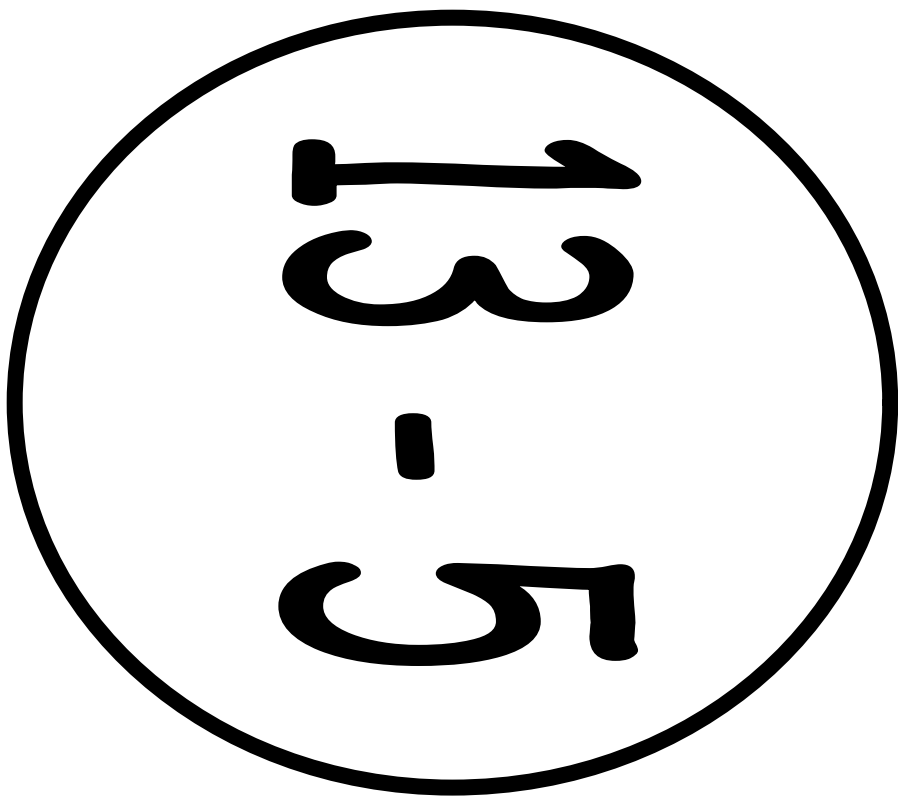
10

45 +



× 10

29 -



X 3

$$27 - \textcircled{8 + 9} \times 1$$

Teacher Resource 46a
Answer Key

Level 1 Answer Key:

Teacher Resource Number	Answer
Teacher Resource 1	6
Teacher Resource 2	9
Teacher Resource 3	5
Teacher Resource 4	15
Teacher Resource 5	11
Teacher Resource 6	12
Teacher Resource 7	7
Teacher Resource 8	5
Teacher Resource 9	4
Teacher Resource 10	5
Teacher Resource 11	12
Teacher Resource 12	25
Teacher Resource 13	1
Teacher Resource 14	8
Teacher Resource 15	17

Level 2 Answer Key

Teacher Resource Number	Answer
Teacher Resource 16	6
Teacher Resource 17	27
Teacher Resource 18	10
Teacher Resource 19	5
Teacher Resource 20	11
Teacher Resource 21	2
Teacher Resource 22	2
Teacher Resource 23	25
Teacher Resource 24	2
Teacher Resource 25	20
Teacher Resource 26	4
Teacher Resource 27	25
Teacher Resource 28	10
Teacher Resource 29	24
Teacher Resource 30	17

Level 3 Answer Key

Teacher Resource Number	Answer
Teacher Resource 31	10
Teacher Resource 32	36
Teacher Resource 33	16
Teacher Resource 34	14
Teacher Resource 35	2
Teacher Resource 36	13
Teacher Resource 37	7
Teacher Resource 38	15
Teacher Resource 39	8
Teacher Resource 40	14
Teacher Resource 41	22
Teacher Resource 42	15
Teacher Resource 43	55
Teacher Resource 44	5
Teacher Resource 45	10

**But what happens
when *all* of those
operations are in
the *same* math
problem?**



**There is a phrase
we use to help us
remember the order
of operations...**



Pardon My Dear Aunt Sally!



Pardon

**1. Solve inside
parentheses ()**



For Example:

$$\begin{array}{r} 10 + (5 - 4) \times 3 \\ 10 + 1 \times 3 \end{array}$$



My Dear
2. Multiply or Divide
from
left to right.



For Example:

$$10 + (5 - 4) \times 3$$

$$10 + 1 \times 3$$

$$10 + 3$$



Aunt Sally!

**2. Add or Subtract
from
left to right.**



For Example:

$$10 + (5 - 4) \times 3$$

$$10 + 1 \times 3$$

$$10 + 3$$

$$13$$



**Now it is your turn
to try!**



What should I solve first in $5 \times 3 - (2 + 2)$?

$$5 \times 3$$

$$3 - 2$$

$$2 + 2$$

$$5 + 2$$



What should I solve first in $6 + 4 \times 3$?

$$4 \times 3$$

$$6 + 4$$

$$6 + 3$$

$$10 \times 3$$



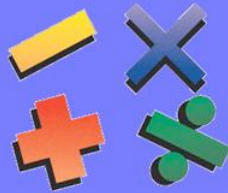
Solve: $(5 + 2) \times 6$

17

42

32

16



Solve: $(20 + 2) - 6$

8

12

10

16



Solve: $48 \div (8 - 2) + 4$

8

12

10

17



Solve: $18 - (6 \times 1) \times 2$

6

4

24

16



Good Job!

**You now know a lot
about the Order of
Operations!**



Order of Operations

Pardon

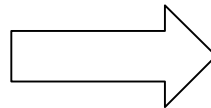
()



My Dear

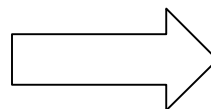
\times or \div

Left to Right



Aunt Sally

$+$ or $-$



Left to Right

BALANCE GAME

Directions:

- 1. Ask students to stand and balance on one foot. (NO HOPPING)**

Generate discussion with the following questions:

- How does it feel to stand on one foot?**
 - How do you know you are balanced?**
 - How do you know you are not balanced?**
- 2. Ask students to balance on the opposite foot.**
 - 3. "Flamingo" - Students balance on one foot while placing the other foot against the opposite knee.**

“Flamingo” Balance Contest: On the signal “go”, students perform the “Flamingo Balance”. If a student removes his/her foot, hops, or any other off-balance movement, he/she must sit down. The remaining one, two, three, etc. students win the contest.

PEOPLE GAME

The purpose of this activity is to allow the students to develop an understanding of odd, even, equal and unequal.

Directions:

1. Using the classroom area, divide the students into two different groups.
2. Divide the students evenly at first to generate a discussion of even and equal.
3. Next, divide the students using an odd number combination. Generate discussion about odd numbers and unequal.

Sample Questions:

Tell me something you know about odd and even numbers?

Can odd numbers be separated evenly?

What must we do to make our class even? Odd?
Equal? Unequal?

Name: _____

Part ASolve the expression below using $>$, $<$, or $=$.

$$18 + 21 \quad \boxed{=} \quad 19 + 20$$

Students receive 1 point for correctly answering part A.**Part B**

Use what you know about equivalent expressions to explain why your answer is correct. Use words, pictures, and/or numbers in your explanation.

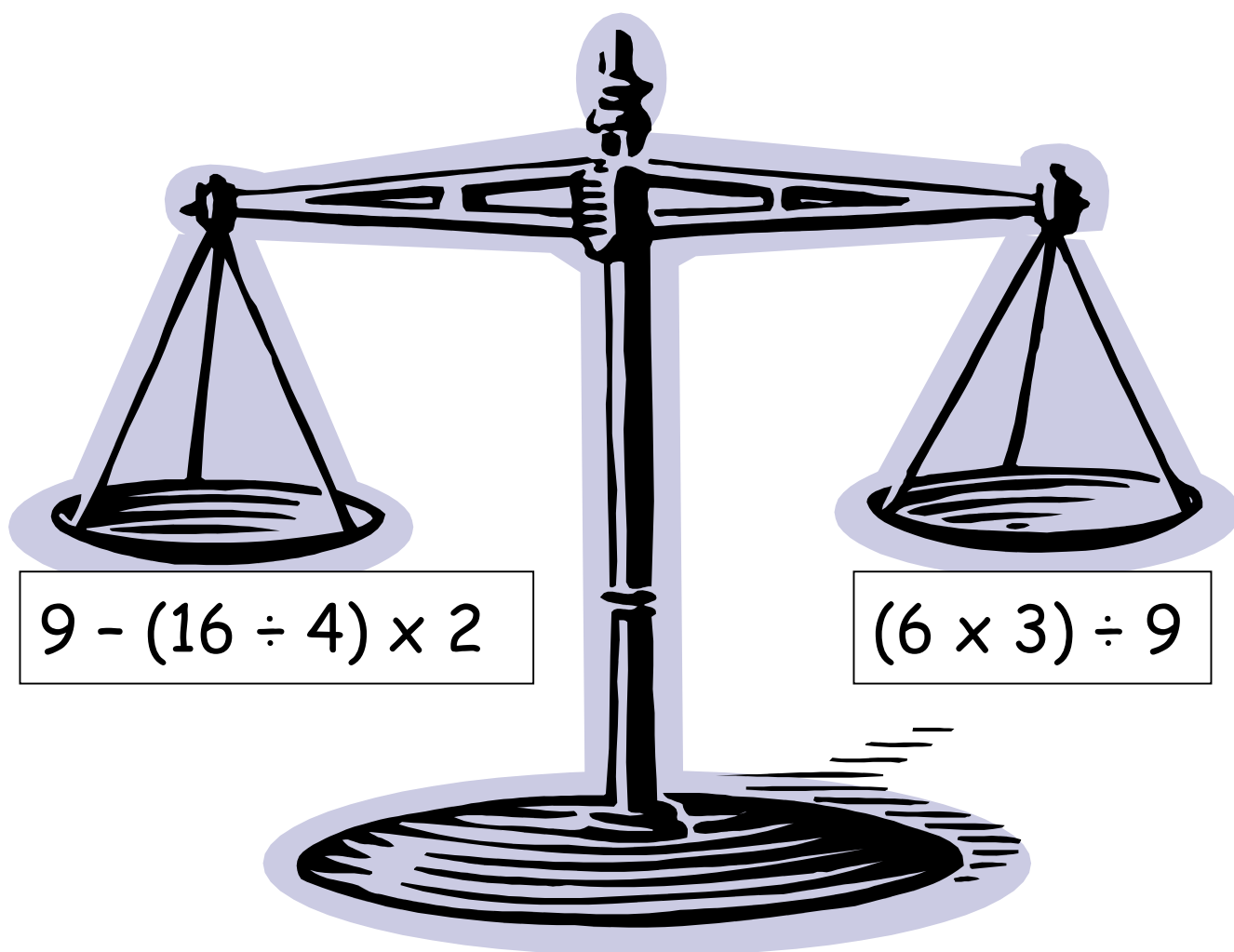
Students receive a possible 2 points**Model Responses 1:**Equivalent is when 2 or more expressions (or number sentences) have the same amount.

$18 + 21 = 39$ and $19 + 20 = 39$

 $39 = 39$ so the expressions are equivalent.**Model Response 2:**Equivalent: Same amount

$18 + 21 = 39$

$19 + 20 = 39$



Teacher Resource 53a
Answer Key





$\begin{array}{c} 8 + (4 \times 5) - 3 \\ 25 \end{array}$	$\begin{array}{c} 1 + (6 \times 5) - 6 \\ 25 \end{array}$
$\begin{array}{c} 18 \div 3 + 2 \\ 8 \end{array}$	$\begin{array}{c} (8 \times 8) - (7 \times 8) \\ 8 \end{array}$
$\begin{array}{c} 15 - 3 \times 3 \\ 6 \end{array}$	$\begin{array}{c} (20 + 16) \div 6 \\ 6 \end{array}$
$\begin{array}{c} 9 + (4 + 5) \times 3 \\ 36 \end{array}$	$\begin{array}{c} 8 \times 2 + 20 \\ 36 \end{array}$
$\begin{array}{c} 19 - (5 \times 3) \div 3 \\ 14 \end{array}$	$\begin{array}{c} 10 + 4 - (5 \times 0) \\ 14 \end{array}$
$\begin{array}{c} 21 - (4 - 2) \\ 19 \end{array}$	$\begin{array}{c} 5 + (7 \times 2) \\ 19 \end{array}$
$\begin{array}{c} 27 - (8 + 9) \times 1 \\ 10 \end{array}$	$\begin{array}{c} 19 - (81 \div 9) \\ 10 \end{array}$

$7 + 3 + (1 \times 1)$ 11	$7 + 6 \times 2 - 8$ 11
$(6 \times 7) - 4$ 38	$101 - (9 \times 7)$ 38
$75 - (12 \div 4)$ 72	$8 + (8 \times 8)$ 72
$8 + (7 \times 0) - 8$ 0	$(5 \div 5) - 1$ 0
$3 + 2 \times 6$ 15	$39 - (6 \times 4)$ 15
$9 + 6 \times 3$ 27	$(6 \times 6) - 9$ 27
$20 + 5 \times 5$ 45	$(5 \times 4) + (5 \times 5)$ 45

$$(7 \times 7) + 3$$
$$52$$

$$58 - 3 \times 2$$
$$52$$

Student Vocabulary Cards

<p>Order of Operations</p> <p>The order in which we solve a mathematical expression</p>  <ol style="list-style-type: none"> 1. Parentheses () 2. Multiplication or Division from left to right 3. Addition or Subtraction from left to right 	<p>Equivalent Equations</p> <p>Two or more expressions that have the same amount.</p> 
<p>Balance</p> <p>The act of having one or more things equal or steady.</p> 	<p>Inequality</p> <p>Two or more expressions that do <u>not</u> have the same amount.</p> 

Operation Answer Key

You are the head doctor at the local hospital. There are many patients waiting to have surgery, but all of the patients' charts got mixed up and no one knows who is supposed to have which surgery.

3. Solve the expressions below on a separate piece of paper.
4. Cut out the expressions and glue them under their **equivalent** expressions to match the patient with his/her correct surgery.

heart
surgery



$$17 - (3 \times 5) + 2$$

lung
surgery



$$2 + 40 \div 8$$

foot
surgery



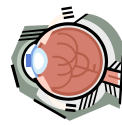
$$6 \times 2 + (3 \times 1)$$

bone
surgery








$$9 - 1 \times 5$$

eye
surgery



$$1 + (6 \times 3)$$

Answer Key:

 $2 + (18 \div 9)$	 $(4 + 3) \times 1$	 $(7 \times 4) - 13$	 $19 + 1 - (4 \times 4)$	 $2 + (5 \times 3) + 2$
--	---	---	--	---

Title: Order of Operation
Dr. X. Pression

Brief Overview:

The purpose of the lesson is for the student to master the concept of the Order of Operations so that an understanding of equivalent expressions is developed. The three-day Algebra lesson begins with the teacher providing review and reinforcement of the Order of Operations through a multi-modal approach to

learning. At the conclusion of the three-day lesson, the students will be able to use Order of Operations to solve expressions and determine what constitutes an equivalent expression.

NCTM Content Standard/National Science Education Standard:

Algebra - Represent and analyze mathematical situations and structures using algebraic symbols: Express mathematical relationships using expressions.

Grade/Level:

Grade 4

Duration/Length:

Three one-hour lessons

Student Outcomes:

Students will:

- Students will be able to identify the Order of Operations
- Students will be able to apply the Order of Operations to solve mathematical expressions.
- Students will be able to demonstrate an understanding of the concept of balance using kinesthetic approaches, real-world objects, and mathematical expressions.
- Students will demonstrate knowledge of the terms: Order of Operations, balance, equivalent expressions, inequality.
- Students will apply their learning of equivalent expressions to a new real-life situation.

Materials and Resources:

- Teacher Resources 1-55
- Student Resources 1-12 (Vocabulary Cards found on Teacher Resource 54)
- Balance Scale
- Classroom objects of teacher's choice. Examples include: stapler, book, glue, scissors, picture frame, weight, etc.
- 2 or 3-Dimensional Geometric Shapes
- PowerPoint Presentation
- Crayons or colored pencils
- Chart paper (1 for every 3-4 students)
- Mini chalkboards, wipe-off boards, or gel boards (1 per pair).
- Connecting Cubes
- Place-value models (flats, rods, units)

- Optional: Doctor's costume, medical equipment

Development/Procedures:

Lesson 1

Pre-assessment

- Distribute Student Resource 1. Students will independently solve two algebraic expressions using the Order of Operations.

Launch

- Provide the students with an explanation of reasons particular tasks are done in a certain order. Real world examples include: people put on socks before shoes, the hitter in baseball must run to first base before second, the letter A comes before the letter B in the alphabet, etc.

Teacher Facilitation

- Apply the explanation of order to math. This is called the Order of Operations. Instruct the concept using mnemonic saying: "Pardon My Dear Aunt Sally," explaining how this dictates the Order of Operations (P in Pardon represents that expressions are solved inside parentheses first; M and D represent that next we solve multiplication and division in the order in which they appear; A and S represent the way we solve addition and subtraction in the order in which they appear).
- Display Teacher Resource 48 of the steps for the Order of Operations. Emphasize that multiplication is not always solved before division. Just as people read from left to right, multiplication and division are solved from left to right. Model an example on the board with the following expression: $18 \div 3 \times 2$. Repeat sample expressions for addition and subtraction using the board to model.
- Students seem to struggle with the concept of solving the expression inside the parentheses first. Therefore, Teacher Resources 1-45, Level 1 – Level 3, requires the student to focus on the enclosed area (i.e., parentheses) to represent the first order of operation. As the students progress through each Level, numbers and symbols appear on one and eventually both sides of the circle, challenging the students to develop the appropriate skills necessary to successfully solve the expression. Note: Level 1 (Teacher Resources 1-15), Level 2 (Teacher Resources 16- 30), Level 3 (Teacher Resources 31- 45)

Student Application

- Distribute Teacher Resources 1-15 for Level 1 throughout the classroom and/or workspace. The emphasis for this learning opportunity is on the incorporation of a multi-modal approach to learning. Therefore, provide

the students with a kinesthetic approach to this concept. Integration: Use Hula Hoops and/or Venn Diagrams Circles.

- Begin by organizing the students in groups of 2-4 students.
(*Differentiate the process for students with special needs by grouping the students heterogeneously).
- On a signal, the students begin at Level 1. The students must successfully solve a designated number of expressions before moving on to the next levels (It is recommended the students solve three or more expressions in order to reinforce the concept).
- Informal assessment occurs as the teacher circulates throughout the workspace monitoring student progress. Consider having students report to the teacher following the completion of each expression in order to ensure success. The teacher will circulate using a clipboard with answer sheet attached (Teacher Resource Level 1, 46a), (Teacher Resource, Level 2, 46b), (Teacher Resource, Level 3, 46c) and colored marker available. The answer sheet is color coded so that the teacher can quickly and efficiently place the color mark on the student worksheet. For example: Level 1 (Black Marker), Level 2 (Green Marker), Level 3, (Red Marker). Other Options: (1) consider integrating lesson with Physical Education teacher; (2) reserve activity room; (3) use outdoor facilities; (4) use cafeteria. Teacher must be willing to differentiate with nature of students
- Students progress to Level 2 using Teacher Resources 16-30 once they are successful at the first level. This involves ensuring the student response is correct. Consider having the students complete at least three worksheets in order to provide for practice and repetition.
- Students progress to Level 3 using Teacher Resources 31 – 45, once they are successful at the second level. This involves ensuring the student response is correct. Consider having the students complete at least three worksheets in order to provide for practice and repetition.
- Engage the class in a discussion of the reasons they solved their problems in a particular order. Use visual representation when necessary. Use the following questions to facilitate this discussion: What did you ask yourself before solving the expression? What was your first step? Why? What did you do next? How did you know that is what you needed to do? Imagine you are the teacher, how would you explain what to do?
- Using Student Resource 2, provide the students with two expressions for them to solve using the Order of Operations (*Differentiate for students who are gifted and talented by allowing them to create their own expression to solve).

Embedded Assessment

- Distribute Student Resource 3 to complete independently.

Re-teaching/Extension

- Consider reteaching by allowing students that are experiencing difficulty to complete the Power Point presentation using Teacher Resources 47a – 47i. (*Differentiate the process for students with decoding difficulties by

reading the PowerPoint aloud to students). Also, consider printing the Power Point in order to provide for additional assistance.

- Consider providing a floor-size expression so that students experience the opportunity to physically move through the Order of Operations.

Additional Resources

- Reference:
Nurnberger-Haag, Julie. "Order of Op Hop." Mathematics Teaching in the Middle School. 2003. Online. The National Council of Teachers of Mathematics. Internet. 26 July 2006. Available: nctm.org.
 - Student website - Order of Operations Game:
<http://www.funbrain.com/algebra/index.html>

Lesson 2

Pre-assessment

- Distribute Student Resource 4. Direct the students to write two number sentences that are equivalent to each other.

Launch

- Follow the directions on Teacher Resources 49a and 49b to engage the students in the "Balance Game." (*Differentiate the process for students with physical disabilities or gross motor difficulties by allowing them to hold on to a walker or desk to assist in balancing).

Teacher Facilitation –

- Engage the students in a discussion about the "Balance Game" and the task they completed. Possible discussion questions are listed on the Balance Game Teacher Resources 49a and 49b.
- Follow the directions on Teacher Resource 50 to play "People."
- Generate a discussion with the students about the following terms: balance, equivalent, odd, even, unequal, etc.
- Demonstrate the term equivalent by using a balance scale and classroom objects to connect to real world observations.

Student Application

- Allow opportunities for the student to experiment with the term equivalent by allowing individuals to place 2 or 3 dimensional shapes on the balance scale. This is a great opportunity to review shapes. Use objects that balance the scale and as non-examples, objects that do not. Discuss with the student things might not look the same but can still be balanced/equal. For example: a person might weigh the same as another person, but they could be taller or shorter.

- Provide a connection for the student by explaining just as objects can be balanced, mathematical expressions can also be balanced. In order to prepare the student for balancing expressions, distribute Student Resource 5. Consider allowing students to use connecting cubes to develop a visual and tactile sense. (*Differentiate the process for students with special needs by providing them with Student Resource 12, a Hundreds Chart, for additional support). If students are observed using the Commutative Property, challenge them to find another number sentence that uses numbers different than their partner's numbers.
- In order to provide a higher level thinking opportunity, allow students to exchange their paper with a partner to develop additional number sentences with different operation signs. Students could create number sentences without the connecting cubes in order to generate equivalent expressions with more difficult numbers (i.e. $56 + 31 = 87$ and $100 - 13 = 87$). Ask students to share orally the different number sentences they have created.
- The review should include reasons why the number sentences are equivalent. Students discuss orally why their number sentences are equivalent.

Embedded Assessment

- Students work independently to complete the BCR Student Resource 6 (*Differentiate the process for students with written language disabilities by providing a scribe). Models of excellence and scoring information are provided on Teacher Resource 51.

Re-teaching/Extension

- Consider providing small group instruction for students experiencing difficulty. A kinesthetic approach includes creating a number line on the floor. The teacher calls out different numbers and students take turns moving to those numbers. The teacher records the numbers on the chalkboard until equivalent expressions are created. Allowing students to say the numbers provides additional interaction.

Additional Resources

- Website - Game to Reinforce Vocabulary: Match the Equivalent sets of Money: http://www.harcourtschool.com/activity/con_math/g03c07.html

Lesson 3

Pre-assessment

- Have students complete Student Resource 7

Launch

- Consider using partner or group activities to engage the students.
- Using chart paper, ask students to generate ideas demonstrating previous knowledge acquired of the terms equivalent, equal, and unequal. (For

example: Tell me what you already know about these terms. What makes something equivalent, equal, unequal? What have we already used in class to demonstrate equivalent, equal, and unequal?)

Teacher Facilitation

- Using mathematical examples, introduce the term, “inequality,” in order to make a mathematical connection to the term unequal. Using connecting cubes, a mathematical example could be: $6 + 5 \neq 9$, $10 + 5 \neq 14$, etc.
- Using the Order of Operations process, model the concept of how two mathematical expressions can be equivalent.
- Refer to Teacher Resource 52 to further demonstrate how expressions can be equivalent. Place the Resource on the chalkboard. Solve both sides using the Order of Operations.

Student Application

- Distribute one expression card from Student Resources 8a, 8b, and 8c to each student. An answer key is provided on Teacher Resources 53a – 53c.
- Direct the students to solve the expression on his/her card.
- When students have solved their expression, have them circulate in the room to find a partner with an equivalent expression.
- Direct students to return to their work area with their partner and cooperatively explain to each other the reasons they know the expressions are equal.
- Discuss the strategies the students used to successfully complete the task above.
- Consider using mini chalkboards, wipe-off boards, or gel boards to have the students work together to generate and solve equivalent expressions.
- Teacher Resource 54 to reinforce/reteach vocabulary.

Embedded Assessment

- Provide the students with Student Resource 9 (*Differentiate the product for students with disabilities in the area of mathematics by allowing them to complete Student Resource 10).
- Teacher Resource 55 (Answer Sheet).

Re-teaching/Extension

- Consider using other types of manipulatives (coins, Cuisenaire rods, and connecting cubes) to reinforce the terms equivalent and balance. Refer to reteaching and resource activities in Lesson 1 for students experiencing difficulty with Order of Operations.

Summative Assessment:

The students will demonstrate their knowledge of Order of Operations, balance, and equivalent expressions by completing Student Resource 11. An average of 80% or better will demonstrate successful mastery of the unit concepts. Consider providing intervention for students receiving less than 60%. (*Differentiate for gifted/talented students by substituting whole numbers for decimals and/or fractions).

Authors:

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Name: _____

Directions: Simplify the expressions below.

2. $(15 \div 3) \times (5 + 4)$

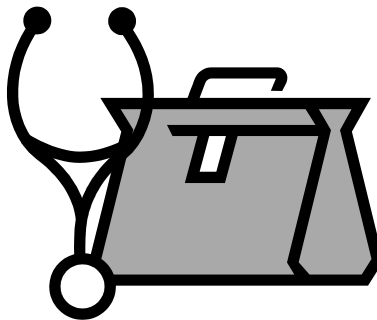
2. $7 \times (4 + 5)$

Name: _____

Directions: Solve. Remember to use the Order of Operations.

$$9 + (8 \times 3) \div 4$$

$$4 \times 3 - (5 \times 2) + 1$$



Name _____

Directions: Solve the expressions below.

1. $(15 \div 3) \times (5 + 4)$

2. $7 \times (4 + 5)$

Name: _____

Directions: Write two number sentences that are equivalent to each other.

1. _____ + _____ = _____

2. _____ + _____ = _____

Name: _____

Directions: Create a number sentence by placing connecting cubes on the mat below. Write the number sentence that the cubes represent.

--	--

_____ = _____

Now use connecting cubes to create an equivalent number sentence. Write the number sentence that the cubes represent.

--	--

_____ = _____

Name: _____

Part A

Solve the expression below using $>$, $<$, or $=$.

$$18 + 21 \quad \square \quad 19 + 20$$

Part B

Use what you know about equivalent expressions to explain why your answer is correct. Use words, pictures and/or numbers in your explanation.

Name: _____

Directions:

6. On the scale below, write two equivalent expressions.



Student Resource 8a
Expression Cards

$8 + (4 \times 5) - 3$	$1 + (6 \times 5) - 6$
$18 \div 3 + 2$	$(8 \times 8) - (7 \times 8)$
$15 - 3 \times 3$	$(20 + 16) \div 6$
$9 + (4 + 5) \times 3$	$8 \times 2 + 20$
$19 - (5 \times 3) \div 3$	$10 + 4 - (5 \times 0)$
$21 + (4 - 2)$	$5 + (7 \times 2)$

$27 - (8 + 9) \times 1$	$19 - (81 \div 9)$
$7 + 3 + (1 \times 1)$	$7 + 6 \times 2 - 8$
$(6 \times 7) - 4$	$101 - (9 \times 7)$
$75 - (12 \div 4)$	$8 + (8 \times 8)$
$8 + (7 \times 0) - 8$	$(5 \div 5) - 1$
$3 + 2 \times 6$	$39 - (6 \times 4)$
$9 + 6 \times 3$	$(6 \times 6) - 9$

$$20 + 5 \times 5$$

$$(5 \times 4) + (5 \times 5)$$

$$(7 \times 7) + 3$$

$$58 - 3 \times 2$$

Student Resource 9
Lesson 3 Post-Assessment

Name: _____

Directions:

7. On the scale below, write two equivalent expressions.
8. Solve the expressions to justify your answer.



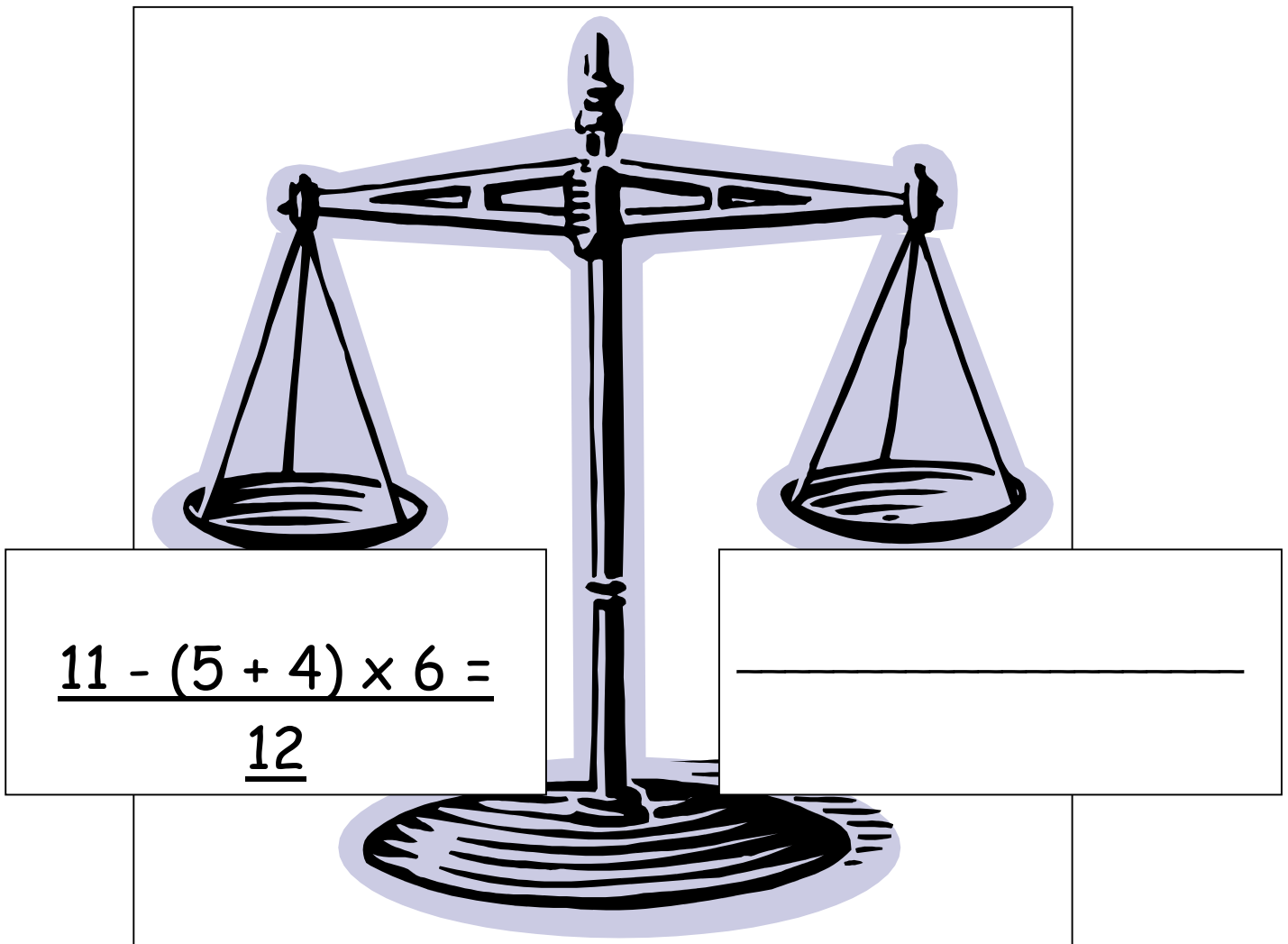
Name: _____

Directions:

9. On the scale below, arrange the numbers and symbols below to create an equivalent expression:

3 7 8 + -

10. Solve the expression to justify your answer.





Dr. _____

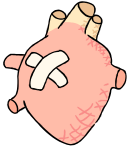
Operation!



You are the head doctor at the local hospital. There are many patients waiting to have surgery, but all of the patients' charts got mixed up and no one knows who is supposed to have which surgery.

5. Solve the expressions below on a separate piece of paper.
6. Cut out the expressions and glue them under their **equivalent** expressions to match the patient with his/her correct surgery.

heart
surgery



$$17 - (3 \times 5) + 2$$

lung
surgery



$$2 + 40 \div 8$$

foot
surgery



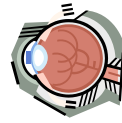
$$6 \times 2 + (3 \times 1)$$

bone
surgery



$$9 - 1 \times 5$$

eye
surgery



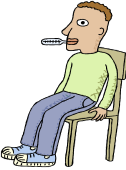




$$1 + (6 \times 3)$$

Glue in the boxes below:

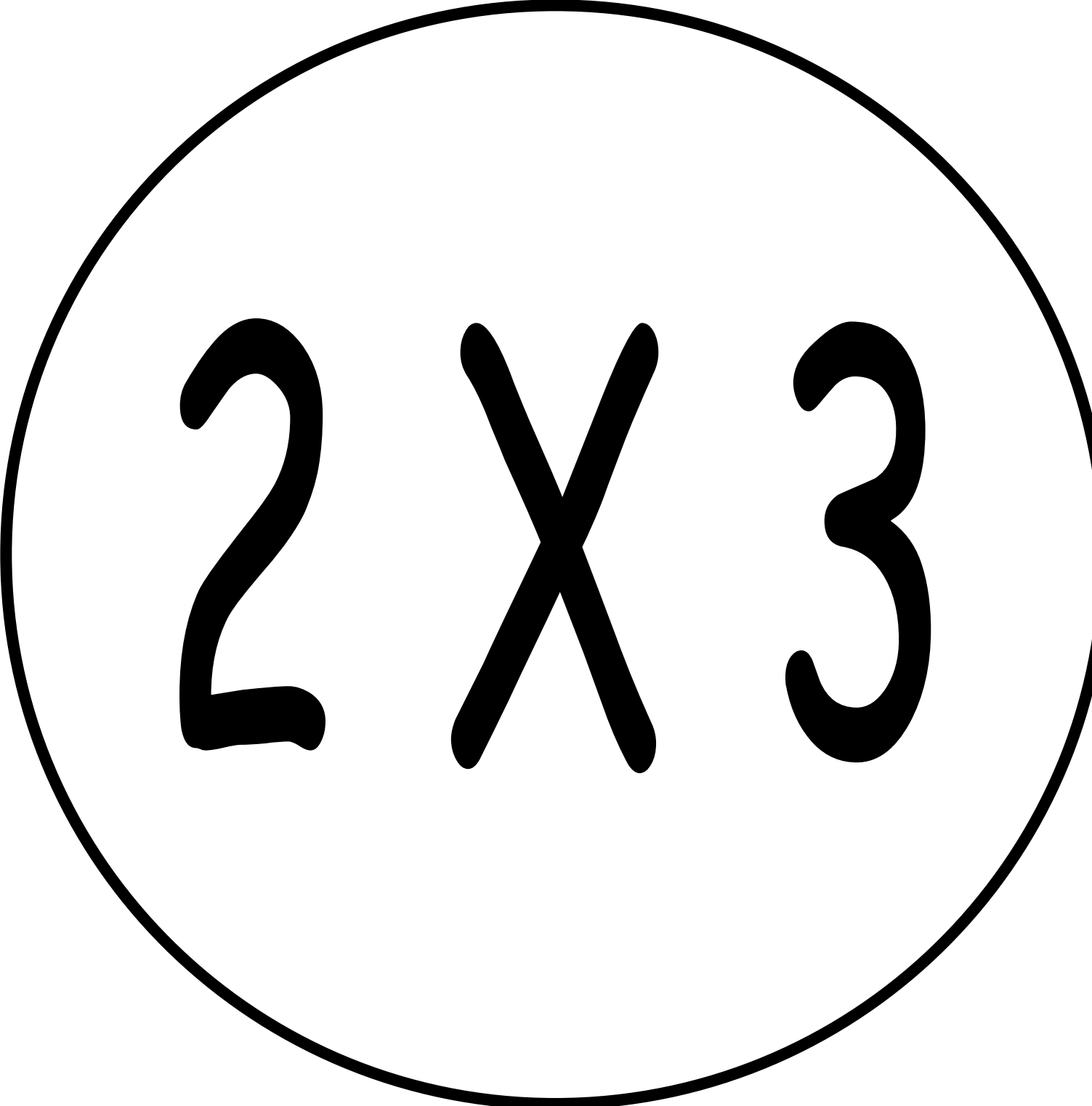
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Cut out the boxes below:

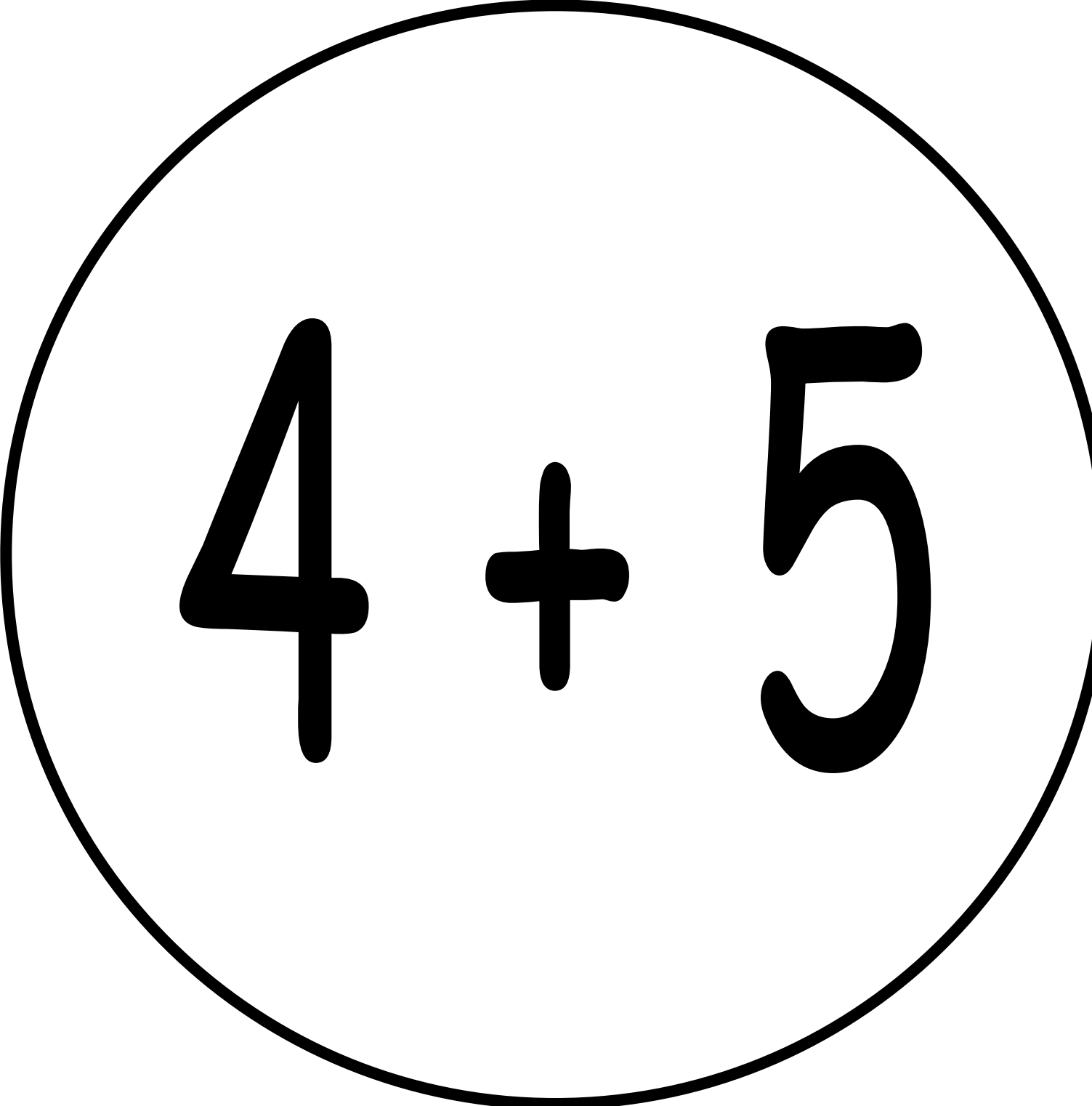
				
$(4 + 3) \times 1$	$2 + (5 \times 3) + 2$	$19 + 1 - (4 \times 4)$	$2 + (18 \div 9)$	$(7 \times 4) - 13$

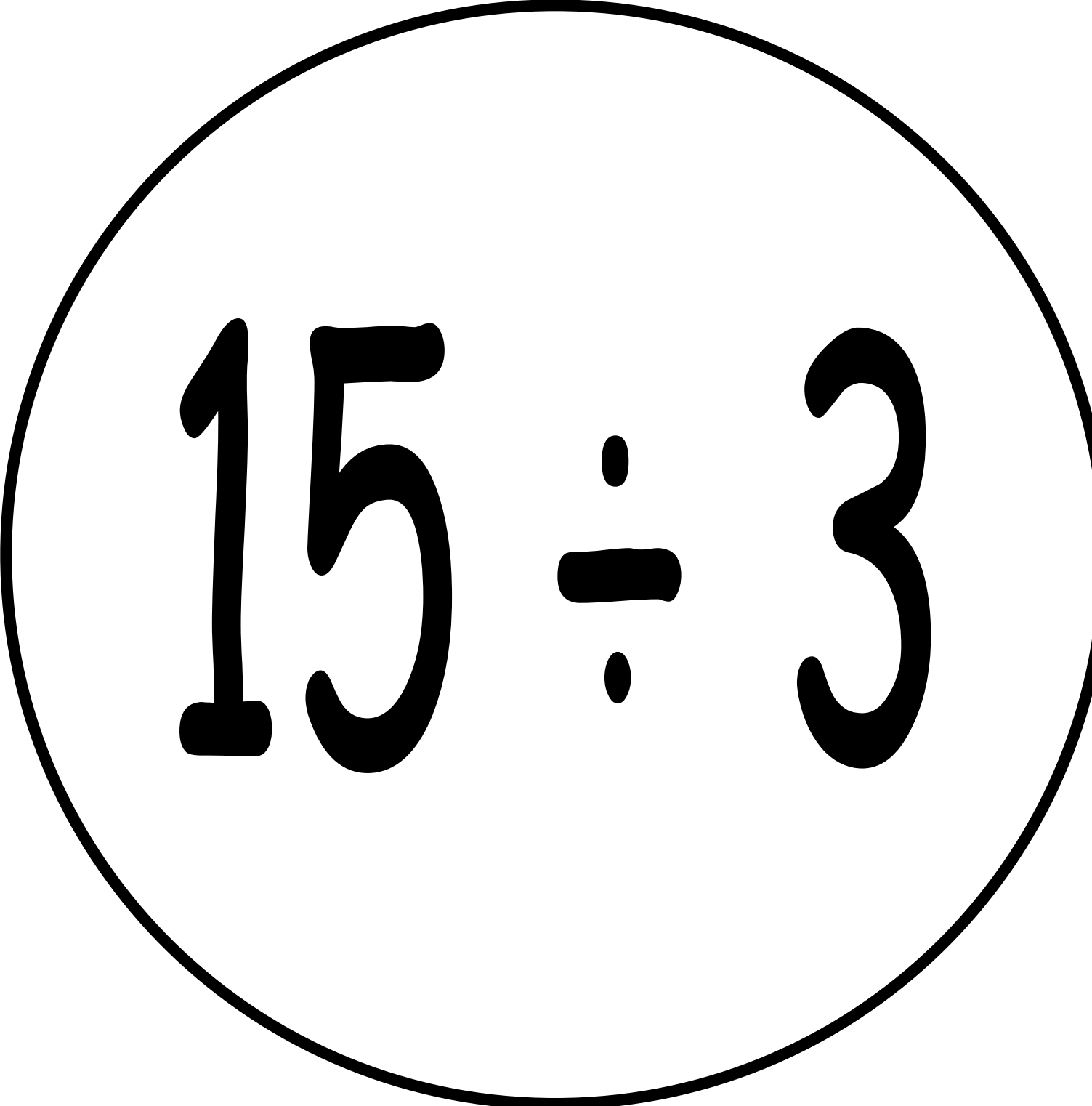
HUNDRED CHART

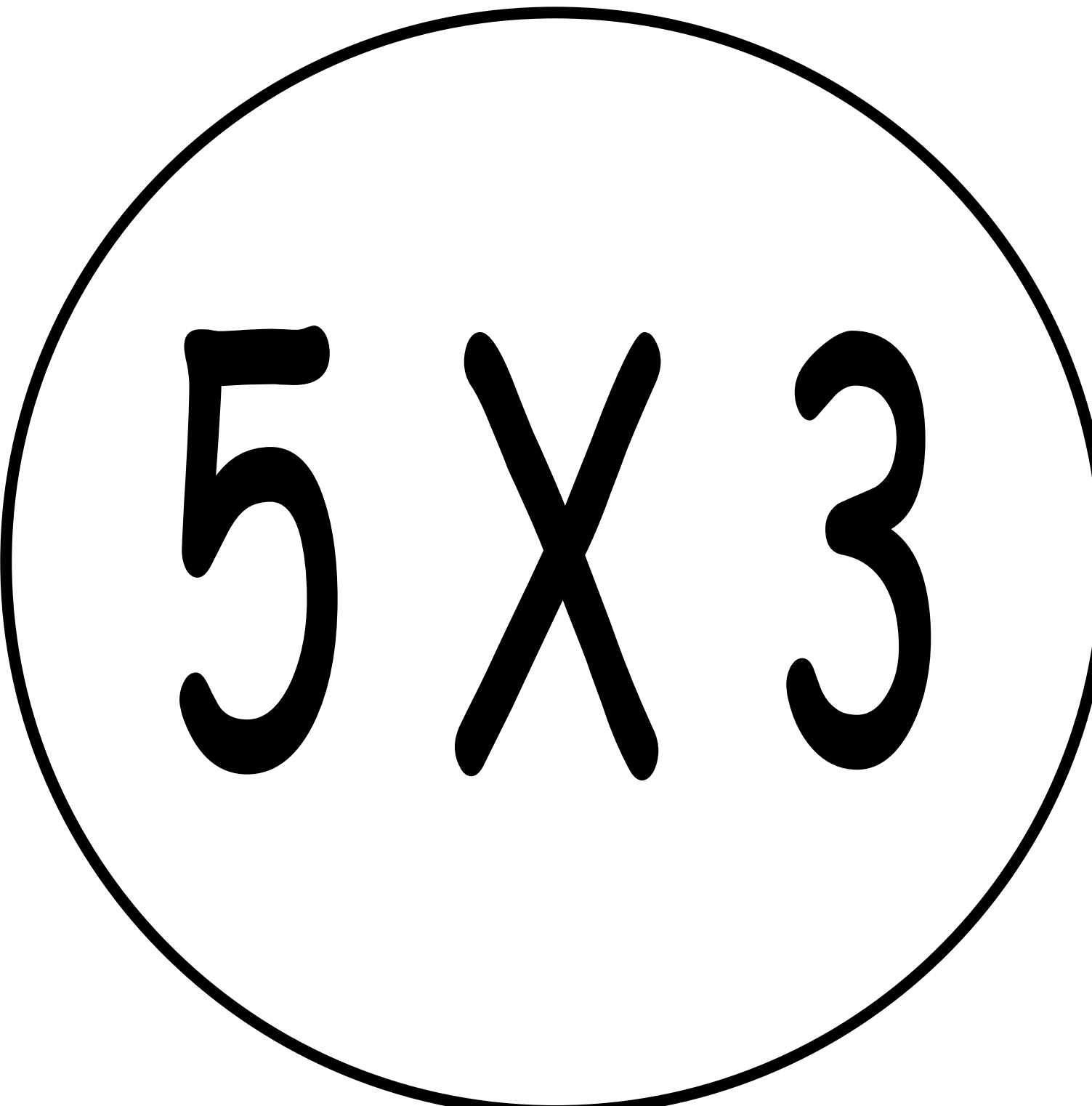
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



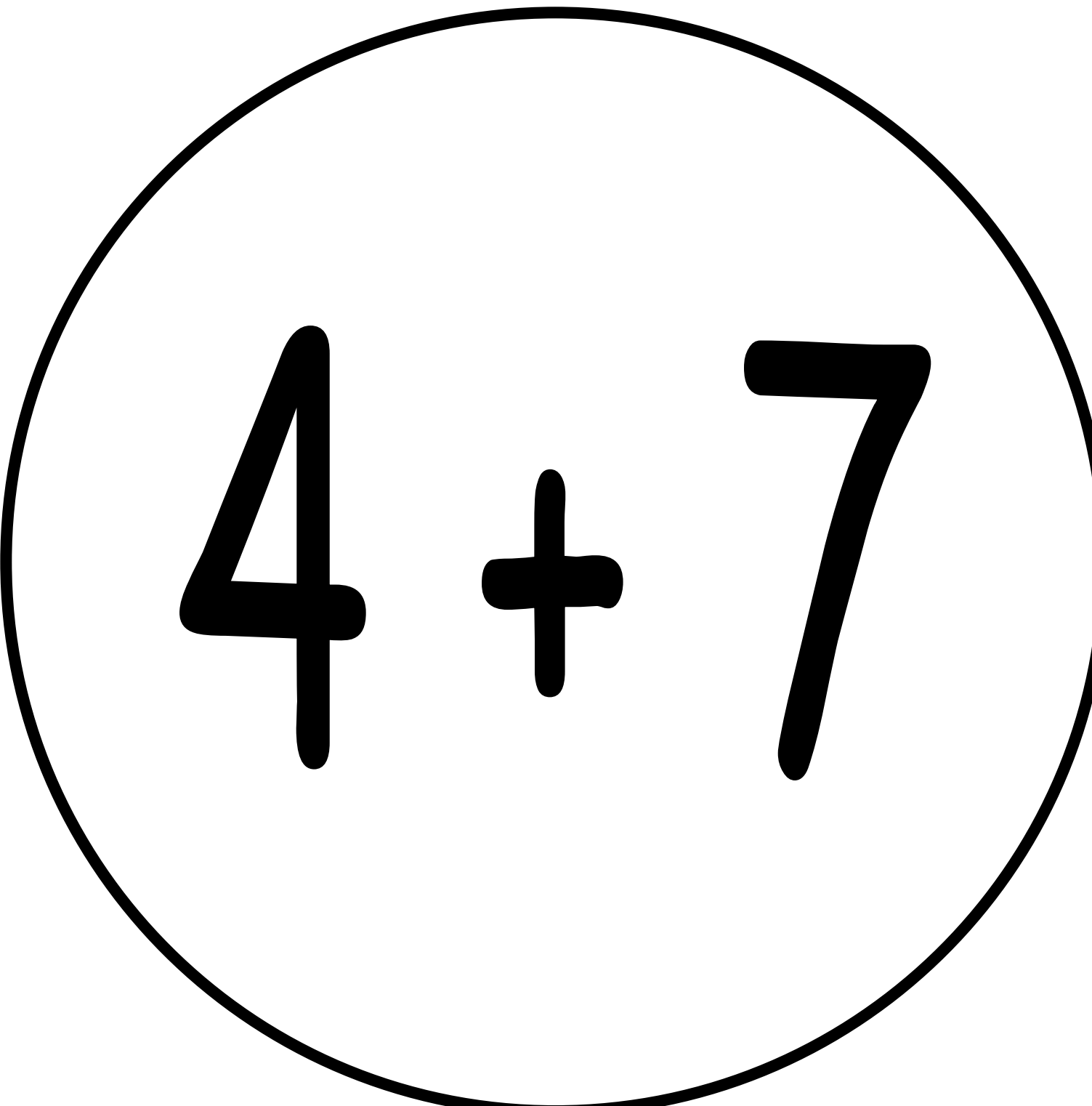
2 x 3

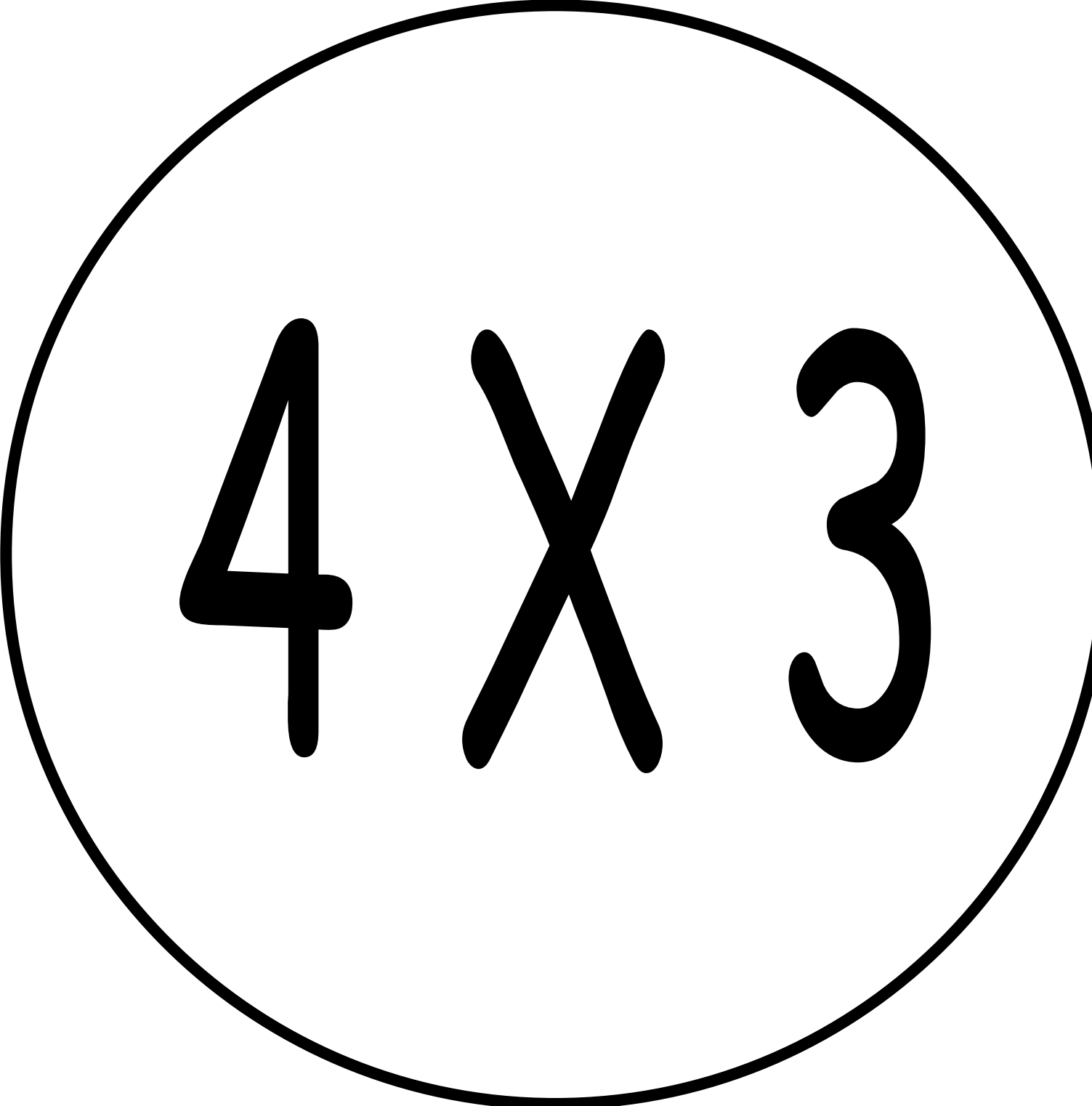

$$4 + 5$$


$$15 \div 3$$



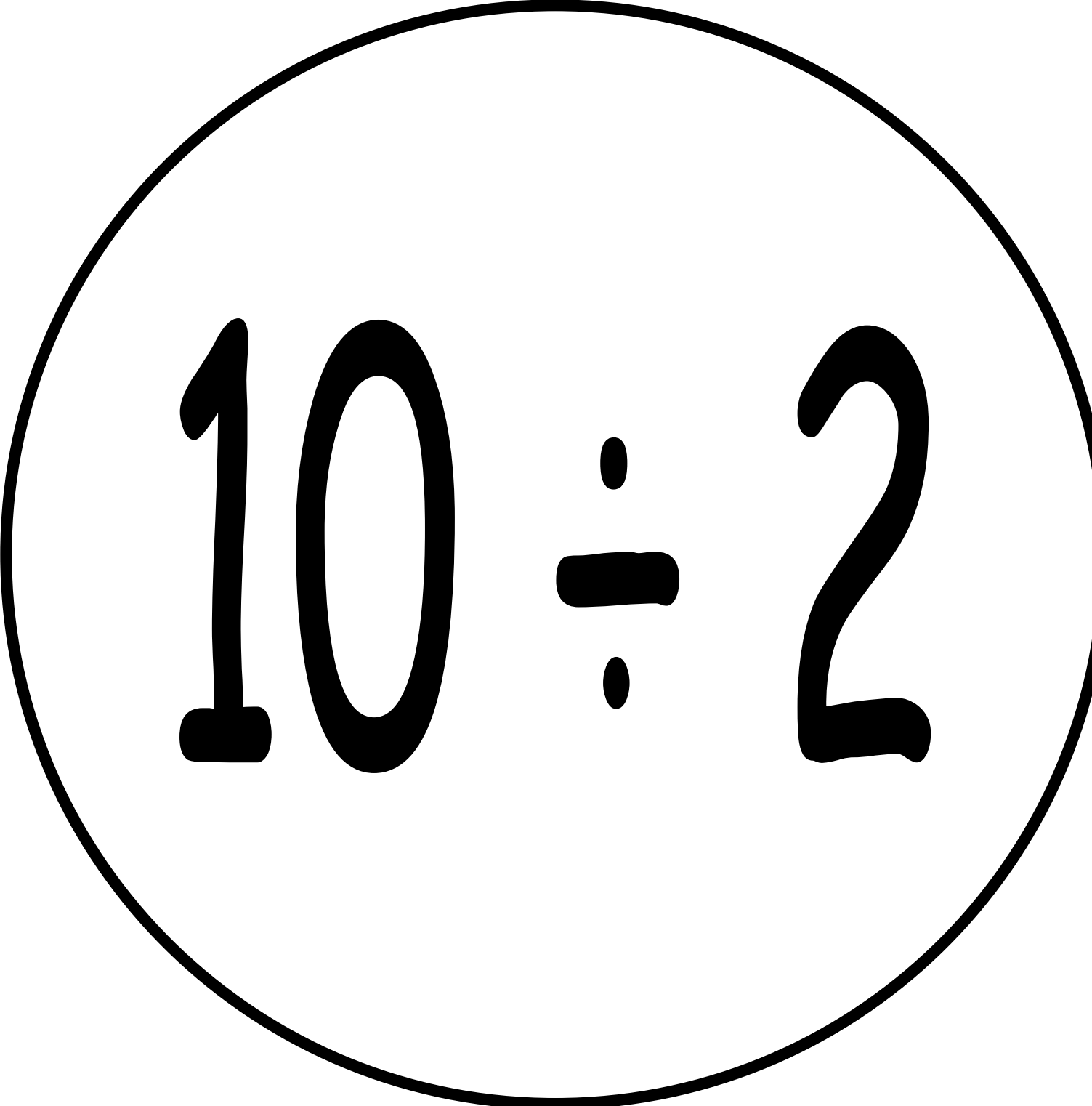
5 x 3


$$4 + 7$$



4 x 3

9-2


$$10 \div 2$$

5 - 1